

Planetary Migration

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Gravitational torques between a planet and gas in the protoplanetary disk result in orbital migration of the planet, and modification of the disk surface density. Migration via this mechanism is likely to play an important role in the formation and early evolution of planetary systems. For masses comparable to those of observed giant extrasolar planets, the interaction with the disk is strong enough to form a gap, leading to coupled evolution of the planet and disk on a viscous time scale (Type II migration). Both the existence of hot Jupiters, and the statistical distribution of observed orbital radii, are consistent with an important role for Type II migration in the history of currently observed systems. We discuss the possibility of improving constraints on migration by including information on the host stars' metallicity, and note that migration could also form a population of massive planets at *large* orbital radii that may be indirectly detected via their influence on debris disks. For lower mass planets with $M_p \sim M_\oplus$, surface density perturbations created by the planet are small, and migration in a laminar disk is driven by an intrinsic and apparently robust asymmetry between interior and exterior torques. Analytic and numerical calculations of this Type I migration are in reasonable accord, and predict rapid orbital decay during the final stages of the formation of giant planet cores. The difficulty of reconciling Type I migration with giant planet formation may signal basic errors in our understanding of protoplanetary disks, core accretion, or both. We discuss physical effects that might alter Type I behavior, in particular the possibility that for sufficiently low masses ($M_p \rightarrow 0$) turbulent fluctuations in the gas surface density dominate the torque, leading to random walk migration of very low mass bodies.

1. Introduction

The extremely short orbital period of 51 Pegasi (Mayor & Queloz 1995) and the other hot Jupiters pose a problem for planet formation, not only because such systems bear little resemblance to the Solar System, but more fundamentally because the high temperatures expected in the protoplanetary disk at radii $a < 0.1$ AU largely preclude the possibility of in situ formation. Disk models by Bell et al. (1997) show that for typical T Tauri accretion rates of $\dot{M} \sim 10^{-8} M_\odot \text{yr}^{-1}$ (Gullbring et al. 1998) the midplane temperature interior to 0.1 AU exceeds 1000 K, destroying ices and, for the very closest in planets, even dust. At least the cores of these hot Jupiters must therefore have formed elsewhere, and subsequently migrated inward. Migration is also likely to have occurred for the larger population of extrasolar planets that now lie within the snow line in their parent disks (Bodenheimer, Hubickyj & Lissauer 2000), though this is a more model-dependent statement since both the location of the snow line (Sasselov & Lecar 2000) and its significance for giant planet formation remain uncertain.

Orbital migration of planets involves a loss of angular momentum to either gas or other solid bodies in the system. Three main mechanisms have been proposed, all of which involve purely gravitational interactions (aerodynamic drag, which is central to the orbital evolution of meter-scale rocks, is negligible for planetary masses). The first is gravitational interaction between the planet and the gas in the protoplanetary disk. This leads to angular momentum exchange between the planet and the gas, and resulting

orbital evolution (Goldreich & Tremaine 1980; Lin, Bodenheimer & Richardson 1996). Since gas giant planets, by definition, formed at an epoch when the protoplanetary disk was still gas-rich, this type of migration is almost unavoidable. It is the main subject of this article. However, further migration could also occur later on, after the gas disk has been dissipated, as a consequence of the gravitational scattering of either planetesimals (Murray et al. 1998) or other massive planets (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997; Papaloizou & Terquem 2001). Some orbital evolution from planetesimal scattering is inevitable, given that the formation of massive planets is highly likely to leave a significant mass of smaller bodies in orbits close enough to feel perturbations from the newly formed giant. In the Solar System, planetesimal scattering could have allowed substantial outward migration (Thommes, Duncan & Levison 1999) of Uranus and Neptune – which have a small fraction of the Solar System’s angular momentum – while simultaneously raising the eccentricities and inclinations of all the giant planets to values consistent with those observed (Tsiganis et al. 2005). Although this is an attractive theory for the architecture of the outer Solar System, invoking a scaled-up version of this process as the origin of the hot Jupiters is problematic. To drive large-scale migration of the typically rather massive planets seen in extrasolar planetary systems would require a comparable mass of planetesimals interior to the initial orbit of the planet. Such a planetesimal disk would in turn imply the prior existence of a rather massive gas disk, which would likely be more effective at causing migration than the planetesimals. Similar reservations apply to models of planet-planet scattering, which is only able to yield a population of planets at small orbital radii if multiple planet formation (with the planets close enough that they are unstable over long periods) is common. That said, the observation that most extrasolar planets have significantly eccentric orbits – which currently defies explanation *except* as an outcome of planet-planet scattering (Ford, Rasio & Yu 2003) – may mean that at least some scattering-driven migration occurs in the typical system.

Figure 1 illustrates how a planet on a circular orbit interacts with the protoplanetary disk. The planet perturbs the gas as it passes by the planet, with angular momentum transport taking place at the locations of resonances in the disk – radii where a characteristic disk frequency is related to the planet’s orbital frequency. For relatively low mass perturbers, the interaction launches a trailing spiral wave in the gas disk, but is not strong enough to significantly perturb the azimuthally averaged surface density profile. In this regime, described as Type I migration, angular momentum transport between the planet and the gas occurs while the planet remains embedded within the protoplanetary disk. The rate of migration is controlled by the sum of the torques arising from the inner and outer Lindblad and corotation resonances, which is generally non-zero (if the sum happened to be close to zero, the planet would act as a source of angular momentum transport in the disk (Goodman & Rafikov 2001) while remaining in place). For the parameters (sound speed, efficiency of angular momentum transport) that are believed to be appropriate for protoplanetary disks, Type I migration occurs for planet masses $M_p \lesssim 0.1 M_J$, where M_J is the mass of Jupiter (Bate et al. 2003), and is most rapid as this critical mass is approached (e.g. Ward 1997). As a result, it is likely to play a particularly important role in the final assembly of giant planet cores.

At higher masses – $M_p \gtrsim 0.1 M_J$ – the angular momentum removal / deposition at the planet’s inner / outer Lindblad resonances is strong enough to repel gas from an annular region surrounding the planet’s orbit, forming a gap in which the surface density is reduced compared to its unperturbed value. For planets of a Jupiter mass and above, the gap is almost entirely evacuated (e.g. the right-hand panel of Figure 1), although mass may continue to flow in a stream across the gap to enter the planet’s Roche

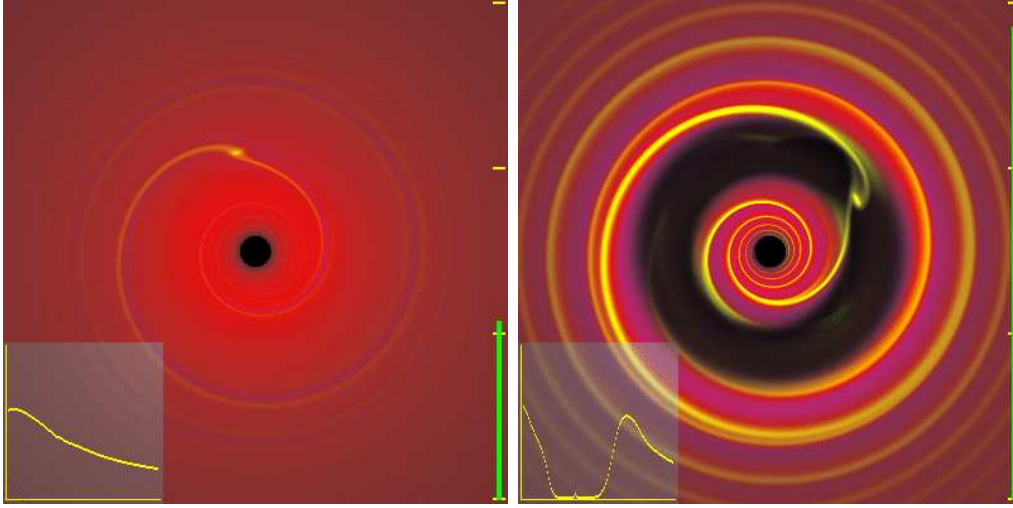


FIGURE 1. An illustration of the interaction between a planet on a fixed circular orbit with a laminar (non-turbulent) protoplanetary disk, computed from a two-dimensional (r, ϕ) hydrodynamic simulation with a locally isothermal equation of state and a constant kinematic viscosity. In the left-hand panel showing the regime of Type I migration, a relatively low mass planet excites a noticeable wave in the disk gas but does not significantly perturb the azimuthally averaged surface density profile (shown as the inset graph). In contrast, a $10 M_J$ planet (right-hand panel) clears an annular gap in the disk, within which the surface density is a small fraction of its unperturbed value. As the disk evolves over a viscous time scale, the planet is predicted to track the motion of the gas (either inward or outward) while remaining within the gap. This is Type II migration. For a movie showing the transition between these regimes, go to http://jilawwww.colorado.edu/~pja/planet_migration.html.

lobe (Artymowicz & Lubow 1996; Lubow, Seibert & Artymowicz 1999). The location of the inner and outer edges of the gap are set by a balance between angular momentum exchange with the planet (which tends to widen the gap), and internal stresses within the protoplanetary disk (‘viscosity’, which tends to close it). Under most circumstances, this balance acts to lock the planet into the long term viscous evolution of the disk gas. At radii where the disk gas is moving inward the planet migrates toward the star, all the while remaining within its gap (Lin & Papaloizou 1986). This is Type II migration, which differs from Type I not only in the presence of a gap, but also because the rate depends directly on the efficiency of angular momentum transport within the protoplanetary disk.

In addition to these well-established migration regimes, qualitatively different behavior may occur at very low masses, and at masses intermediate between the Type I and Type II regimes. At sufficiently low masses (probably of $\sim 10 M_\oplus$ and below) the persistent Type I torque may be overwhelmed by *random* torques from surface density perturbations in a turbulent protoplanetary disk (Nelson & Papaloizou 2004; Laughlin, Steinacker & Adams 2004). This process, which is similar to the heating of Galactic stars by transient spiral arms (Carlberg & Sellwood 1985), may lead to rapid random walk migration on top of Type I drift. This could have important consequences for both core accretion (Rice & Armitage 2003) and terrestrial planet formation. Another uncertain regime lies at the transition mass ($M_p \approx 0.1 M_J$) between Type I and Type II migration, where a partial gap exists and corotation torques can be highly significant. Masset & Papaloizou (2003) suggested that the corotation torques could drive an instability in the *direction* of migration, which if confirmed would be extremely important for massive

planet formation. Subsequent higher resolution simulations by D’Angelo, Bate & Lubow (2005) have demonstrated that extremely high resolution is needed in order to accurately capture torques arising from within and around the Hill sphere. For a Saturn mass planet, the resolution had to be increased to approximately 50 grid zones per Hill radius before a numerically converged solution for the migration rate was attained. Lower resolutions yielded artificially rapid migration. D’Angelo, Bate & Lubow (2005) conclude from this that although torques from the near-planet region are significant, they do not appear to dramatically increase the migration rate.

2. Type I migration

In the Type I regime the perturbation induced by the planet in the gas disk remains small, and the net torque has surprisingly little dependence on the microphysics of the protoplanetary disk (e.g. Goldreich & Tremaine 1978; Lin & Papaloizou 1979). In particular, viscosity – normally the most uncertain element of a protoplanetary disk model – enters only indirectly via its influence on the magnitude and radial gradient of the surface density and sound speed. Generically, the net torque scales with the planet mass as $T \propto M_p^2$, so that the migration time scale at a given radius scales as $\tau \propto M_p^{-1}$. Type I migration therefore becomes increasingly important as the planet mass increases, and is fastest just prior to gap opening (the onset of which *does* depend on the disk viscosity). Despite its attractive lack of dependence on uncertain disk physics, the actual calculation of the net torque is technically demanding, and substantial improvements have been made only recently. Here, we summarize a few key results – the reader is directed to the original papers (primarily by Artymowicz, Ward, and their collaborators) for full details of the calculations.

2.1. Analytic calculations

The simplest calculation of the Type I torque (Goldreich & Tremaine 1978, 1979, 1980) neglects significant pressure effects in the disk close to the planet, and is therefore valid for low m resonances. In this approximation, Lindblad resonances occur at radii in the disk where the epicyclic frequency κ is an integral multiple m of the angular velocity in a frame rotating with the planet at angular velocity Ω_p . For a Keplerian disk, this condition,

$$D(r) \equiv \kappa^2 - m^2 (\Omega - \Omega_p)^2 = 0,$$

can be simplified using the fact that $\kappa = \Omega$. The resonances lie at radii,

$$r_L = \left(1 \pm \frac{1}{m}\right)^{2/3} r_p$$

where r_p is the planet’s orbital radius. The lowest order resonances lie at $r = 1.587r_p$ and $r = 0.630r_p$, but an increasingly dense array of high m resonances lie closer to the planet. Resonances at $r < r_p$ add angular momentum to the planet, while those at $r > r_p$ remove angular momentum. The torque at each resonance T_m can be evaluated in terms of a *forcing function* Ψ_m as,

$$T_m = -\pi^2 m \Sigma \frac{\Psi_m^2}{r dD/dr},$$

where Σ is the gas surface density. Explicit expressions for Ψ_m are given by Goldreich & Tremaine (1979). The net torque is obtained by evaluating the torque at each resonance, and then summing over all m .

For Type I migration the behavior of gas close to the planet, where Ψ_m is largest, is critical. Accurately treating this regime requires elaborations of the basic Goldreich & Tremaine (1979) approach. For a razor thin two-dimensional disk model, - the effects of radial pressure and density gradients, the calculation is described in Ward (1997), and references therein (especially Ward 1988; Artymowicz 1993a, 199b; Korycansky & Pollack 1993). These papers include the shifts in the location of Lindblad resonances due to both radial and azimuthal pressure gradients, which become significant effects at high m . For Lindblad resonances, the result is that the dominant torque arises from wavenumbers $m \simeq r_p/h$, where h , the vertical disk scale height, is given in terms of the local sound speed c_s by $h = c_s/\Omega_p$. The fractional net torque $2|T_{\text{inner}} + T_{\text{outer}}|/(|T_{\text{inner}}| + |T_{\text{outer}}|)$ can be as large as 50% (Ward 1997), with the outer resonances dominating and driving rapid inward migration. Moreover, the small shifts in the locations of resonances that occur in disks with different radial surface density profiles conspire so that the net torque is only weakly dependent on the surface density profile. This means that the predicted rapid inward migration occurs for essentially any disk model in which the sound speed decreases with increasing radius (Ward 1997). Corotation torques - which vanish in the oft-considered disk models with $\Sigma \propto r^{-3/2}$ - can alter the magnitude of the torque but are not sufficient to reverse the sign (Korycansky & Pollack 1993; Ward 1997).

The observation that the dominant contribution to the total torque comes from gas that is only $\Delta r \simeq h$ away from the planet immediately implies that a two-dimensional representation of the disk is inadequate, even for protoplanetary disks which are geometrically thin by the usual definition ($h/r < 0.1$, so that pressure gradients, which scale as order $(h/r)^2$, are sub-percent level effects). Several new physical effects come into play in a three-dimensional disk:

1. The perturbing potential has to be averaged over the vertical thickness of the disk, effectively reducing its strength for high m resonances (Miyoshi et al. 1999).
2. The variation of the scale height with radius decouples the radial profile of the midplane density from that of the surface density.
3. Wave propagation in three-dimensional disks is fundamentally different from that in two dimensions, if the vertical structure of the disk departs from isothermality (Lubow & Ogilvie 1998; Bate et al. 2002).

Tanaka, Takeuchi & Ward (2002) have computed the interaction between a planet and a three-dimensional isothermal disk, including the first two of the above effects. Both Lindblad and corotation torques were evaluated. They find that the net torque is reduced by a factor of 2-3 as compared to a corresponding two-dimensional model, but that migration remains inward and is typically rapid. Specifically, defining a local migration time scale via,

$$\tau = \frac{r_p}{-\dot{r}_p},$$

Tanaka, Takeuchi & Ward (2002) find that for a disk in which $\Sigma \propto r^{-\beta}$,

$$\tau = (2.7 + 1.1\beta)^{-1} \frac{M_*}{M_p} \frac{M_*}{\Sigma r_p^2} \left(\frac{c_s}{r_p \Omega_p} \right)^2 \Omega_p^{-1}.$$

As expected, the time scale is inversely proportional to the planet mass and the local surface density. Since the bracket is $\sim (h/r_p)^2$, the time scale also decreases quite rapidly for thinner disks, reflecting the fact that the peak torque arises from closer to the planet as the sound speed drops.

Although still limited by the assumption of isothermality, the above expression represents the current ‘standard’ estimate of the Type I migration rate. Although slower than

two-dimensional estimates, it is still rapid enough to pose a potential problem for planet formation via core accretion. There is therefore interest in studying additional physical effects that might reduce the rate further. The influence of disk turbulence is discussed more fully in the next Section; here we list some of the other effects that might play a role:

1. **Realistic disk structure models.** The run of density and temperature in the midplane of the protoplanetary disk is not a smooth power-law, due to sharp changes in opacity and, potentially, the efficiency of angular momentum transport (Gammie 1996). Menou & Goodman (2004) have calculated Type I rates in Shakura-Sunyaev type disk models, and find that even using the standard Lindblad torque formula there exist regions of the disk where the migration rate is locally slow. Such zones could be preferred sites of planet formation.

2. **Thermal effects.** Jang-Condell & Sasselov (2005) find that the dominant non-axisymmetric thermal effect arises from changes to the stellar illumination of the disk surface in the vicinity of the planet. This effect is most important at large radii, and can increase the migration time scale by up to a factor of 2 at distances of a few AU.

3. **Wave reflection.** The standard analysis assumes that waves propagate away from the planet, and are dissipated before they reach boundaries or discontinuities in disk properties that might reflect them back toward the planet. Tanaka, Takeuchi & Ward (2002) observe that reflection off boundaries has the potential to substantially reduce the migration rate. We note, however, that relaxation of vertical isothermality will probably lead to wave dissipation in the disk atmosphere within a limited radial distance (Lubow & Ogilvie 1998), and thereby reduce the possibility for reflection.

4. **Accretion.** Growth of a planet during Type I migration is accompanied by a non-resonant torque, which has been evaluated by Nelson & Benz (2003a). If mass is able to accrete freely on to the planet, Bate et al. (2003) find from three-dimensional simulations that $\dot{M}_p \propto M_p$ for $M_p \lesssim 10 M_\odot$, with a mass doubling time that is extremely short (less than 10^3 yr). In reality, it seems likely the planet will be unable to accept mass at such a rapid rate, so the mass accretion rate and resulting torque will then depend on the planet structure.

5. **Magnetic fields.** The dominant field component in magnetohydrodynamic disk turbulence initiated by the magnetorotational instability is toroidal (Balbus & Hawley 1998). Terquem (2003) finds that gradients in plausible toroidal magnetic fields can significantly alter the Type I rate, and in some circumstances (when the field decreases rapidly with r) stop migration. More generally, a patchy and variable toroidal field might lead to rapid variations in the migration rate. Whether this, or density fluctuations induced by turbulence, is the primary influence of disk fields is unclear.

6. **Multiple planets.** The interaction between multiple planets has not been studied in detail. Thommes (2005) notes that low mass planets, which would ordinarily suffer rapid Type I migration, can become captured into resonance with more massive bodies that are themselves stabilized against rapid decay by a gap. This may be important for understanding multiple planet formation (and we have already noted that there is circumstantial evidence that multiple massive planet formation may be common), though it does not explain how the *first* planet to form can avoid rapid Type I inspiral.

7. **Disk Eccentricity.** Type I migration in an axisymmetric disk is likely to damp planetary eccentricity. However, it remains possible that the protoplanetary disk itself might be spontaneously unstable to development of eccentricity (Ogilvie 2001). Papaloizou (2002) has shown that Type I migration can be qualitatively altered, and even reversed, if the background flow is eccentric.

2.2. Numerical simulations

Hydrodynamic simulations of the Type I regime within a shearing sheet geometry have been presented by Miyoshi et al. (1999), and in cylindrical geometry by D’Angelo, Henning & Kley (2002), D’Angelo, Kley & Henning (2003), and Nelson & Benz (2003b), with the latter paper focusing on the transition between Type I and Type II behavior. The most comprehensive work to date is probably that of Bate et al. (2003), who simulated in three dimensions the interaction with the disk of planets in the mass range $1 M_{\oplus} \leq M_p \leq 1 M_J$. The disk model had $h/r = 0.05$, a Shakura-Sunyaev (1973) α parameter $\alpha = 4 \times 10^{-3}$, and a fixed locally isothermal equation of state (i.e. $c_s = c_s(r)$ only). This setup is closely comparable to that assumed in the calculations of Tanaka, Takeuchi & Ward (2002), and very good agreement was obtained between the simulation results and the analytic migration time scale. Based on this, it seems reasonable to conclude that *within the known limitations imposed by the restricted range of included physics*, current calculations of the Type I rate are technically reliable. Given this, it is interesting to explore the consequences of rapid Type I migration for planet formation itself.

2.3. Consequences for planet formation

The inverse scaling of the Type I migration time scale with planet mass means that the most dramatic effects for planet formation occur during the growth of giant planet cores via core accretion (Mizuno 1980). In the baseline calculation of Pollack et al. (1996), which does not incorporate migration, runaway accretion of Jupiter’s envelope is catalyzed by the slow formation of a $\approx 20 M_{\oplus}$ core over a period of almost 10 Myr. This is in conflict with models by Guillot (2004), which show that although Saturn has a core of around $15 M_{\oplus}$, Jupiter’s core is observationally limited to at most $\approx 10 M_{\oplus}$, leading to discussion at this meeting of several ways to reduce the theoretically predicted core mass. Irrespective of the uncertainties, however, it seems inevitable that planets forming via core accretion pass through a relatively slow stage in which the growing planet has a mass of $5 - 10 M_{\oplus}$. This stage is vulnerable to Type I drift.

Figure 2 shows the migration time scale for a $10 M_{\oplus}$ planet within gas disks with surface density profiles of $\Sigma \propto r^{-1}$ (very roughly that suggested by theoretical models, e.g. Bell et al. 1997) and $\Sigma \propto r^{-3/2}$ (the minimum mass Solar Nebula profile of Weiden-schilling 1977). We consider disks with integrated gas masses (out to 30 AU) of $0.01 M_{\odot}$ and $1 M_J$. The latter evidently represents the absolute minimum gas mass required to build Jupiter or a typical extrasolar giant planet. The torque formula of Tanaka, Takeuchi & Ward (2002) is used to calculate the migration time scale τ . As is obvious from the Figure, migration from 5 AU on a time scale of 1 Myr – significantly less than either the typical disk lifetime (Haisch, Lada & Lada 2001) or the duration of the slow phase of core accretion – is inevitable for a core of mass $10 M_{\oplus}$, even if there is only a trace of gas remaining at the time when the envelope is accreted. For more reasonable gas masses, the typical migration time scale at radii of a few AU is of the order of 10^5 yr. Another representation of this is to note that in the giant planet forming region, we predict significant migration ($\tau = 10$ Myr) for masses $M_p \gtrsim 0.1 M_{\oplus}$, and rapid migration ($\tau = 1$ Myr) for $M_p \gtrsim 1 M_{\oplus}$. We can also plot, for the same disk model, an estimate of the isolation mass (Lissauer 1993; using the gas to planetesimal surface density scaling of Ida & Lin 2004a). The isolation mass represents the mass a growing planet can attain by consuming only those planetesimals within its feeding zone – as such it is reached relatively rapidly in planet formation models. Outside the snow line, the migration time scale for a planet (or growing core) at the isolation mass is typically a few Myr, reinforcing the conclusion that migration is inevitable in the early stage of giant planet formation. By contrast, in

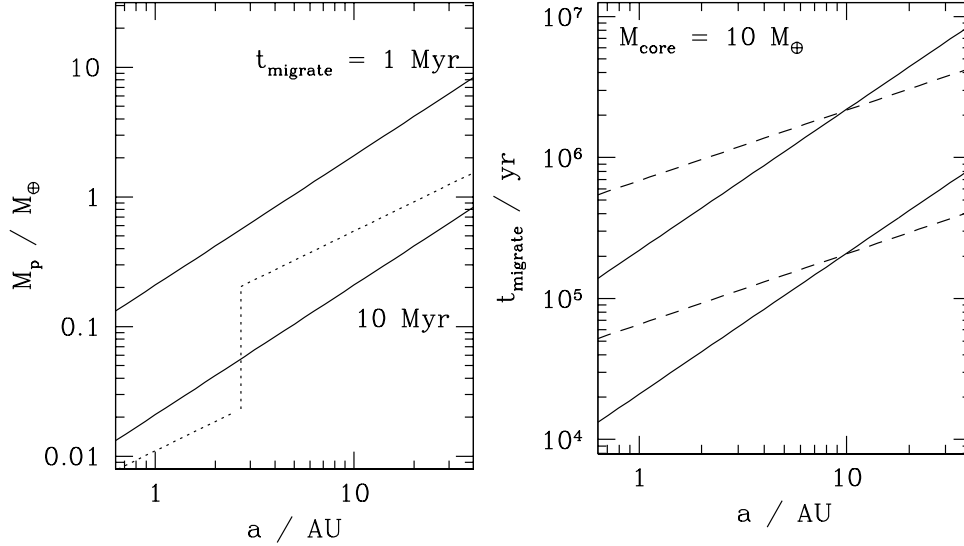


FIGURE 2. Left hand panel: planet mass for which the Type I migration time scale at different radii equals 1 Myr (upper solid curve) and 10 Myr (lower curve). The gas disk is assumed to have a surface density profile $\Sigma \propto r^{-3/2}$, and a mass within 30 AU of $0.01 M_\odot$. The dotted line shows an estimate of the isolation mass in the same disk model, assuming Solar metallicity and a snow line at 2.7 AU. Right hand panel: the migration time scale for a $10 M_\oplus$ core in protoplanetary disks with surface density profiles of $\Sigma \propto r^{-3/2}$ (solid lines) and $\Sigma \propto r^{-1}$ (dashed lines). For each model, the lower curve shows the time scale for a disk with a gas mass of $0.01 M_\odot$ within 30 AU, while the upper curve shows results for an absolute minimum mass gas disk with only $1 M_J$ within 30 AU.

the terrestrial planet region, interior to the snow line, planets need to grow significantly beyond isolation before rapid migration ensues. It is therefore possible for the early stages of terrestrial planet formation to occur in the presence of gas with only limited Type I migration, while the final assembly of terrestrial planets happens subsequently in a gas poor environment.

Does Type I migration help or hinder the growth of giant planets via core accretion? This question remains open, despite a history of investigations stretching back at least as far as papers by Hourigan & Ward (1984) and Ward (1989). Two competing effects are at work:

1. A migrating core continually moves into planetesimal-rich regions of the disk that have not been depleted by the core's prior growth. This depletion is, in part, responsible for the slow growth of Jupiter in static core calculations. Calculations suggest that a rapidly migrating core can capture of the order of 10% of the planetesimals it encounters (Tanaka & Ida 1999), with the collision fraction increasing with migration velocity. Slow migration velocities allow for inward shepherding of the planetesimals rather than capture, and a low accretion rate (Ward & Hahn 1995; Tanaka & Ida 1999).

2. A migrating core must reach the critical core mass before it is lost to the star, on a time scale that, as we noted above, can be an order of magnitude or more smaller than the gas disk lifetime. Unfortunately, a high accretion rate of planetesimals increases the critical core mass needed before runaway gas accretion starts and, at fixed accretion rate, the critical mass also increases as the core moves inward (Papaloizou & Terquem

1999). Migration therefore favors a high rate of planetesimal accretion, but often hinders attaining the critical core mass needed for envelope accretion.

Calculations of giant planet formation including steady core migration have been presented by several groups, including Papaloizou & Terquem (1999), Papaloizou & Larwood (2000), Alibert, Mordasini & Benz (2004) and Alibert et al. (2005). The results suggest that, for a single growing core, Type I migration at the standard rate of Tanaka, Takeuchi & Ward (2002) is simply too fast to allow giant planet formation to occur across a reasonable range of radii in the protoplanetary disk. Most cores are lost to the star or, if they manage to accrete envelopes at all, do so at such small radii that their ultimate survival is doubtful. More leisurely migration, on the other hand, at a rate suppressed from the Tanaka, Takeuchi & Ward (2002) value by a factor of 10 to 100, *helps* core accretion by mitigating the depletion effect that acts as a bottleneck for a static core (Alibert et al. 2005).

This difficulty in reconciling our best estimates of the Type I migration rate with core accretion signals that something is probably wrong with one or both of these theories. Three possibilities suggest themselves. First, the Type I migration rate may be a substantial overestimate, by an order of magnitude or more. If so, there is no need for substantial changes to core accretion theory or to protoplanetary disk models. We have already enumerated a list of candidate physical reasons for why the Type I rate may be wrong, though achieving a sufficiently large suppression does not seem to be straightforward. Second, angular momentum transport may be strongly suppressed in the giant planet formation region by the low ionization fraction, which suppresses MHD instabilities that rely on coupling between the gas and the magnetic field (Gammie 1996; Sano et al. 2000). An almost inviscid disk could lower the threshold for gap opening sufficiently far that the slow stage of core accretion occurred in a gas-poor environment (elements of such a model have been explored by Rafikov 2002; Matsumara & Pudritz 2005). Finally, and perhaps most attractively, it may be possible to find a variant of core accretion that is compatible with undiluted Type I migration. For a single core, we have studied simple models in which random walk migration leads to large fluctuations in the planetesimal accretion rate and an early onset of criticality (Rice & Armitage 2003). In the more realistic situation where multiple cores are present in the disk, it is possible that the early loss of the first cores to form (at small radii just outside the snow line) could evacuate the inner disk of planetesimals, allowing subsequent cores to reach their critical mass and accrete envelopes as they migrate inward. Further work is needed to explore such scenarios quantitatively.

3. Stochastic migration in turbulent disks

To a first approximation the efficiency of angular momentum transport (unless it is very low) has little impact on the predicted Type I migration rate. This assumes, however, that the disk is laminar. More realistically, angular momentum transport itself derives from turbulence, which is accompanied by a spatially and temporally varying pattern of density fluctuations in the protoplanetary disk. These fluctuations will exert *random* torques on planets of any mass embedded within the disk, in much the same way as transient spiral features in the Galactic disk act to increase the velocity dispersion of stellar populations (Carlberg & Sellwood 1985). If we assume that the random torques are uncorrelated with the presence of a planet, then the random torques' linear scaling with planet mass will dominate over the usual Type I torque (scaling as M_p^2) for sufficiently low masses. The turbulence will then act to increase the velocity dispersion of collisionless bodies, or, in

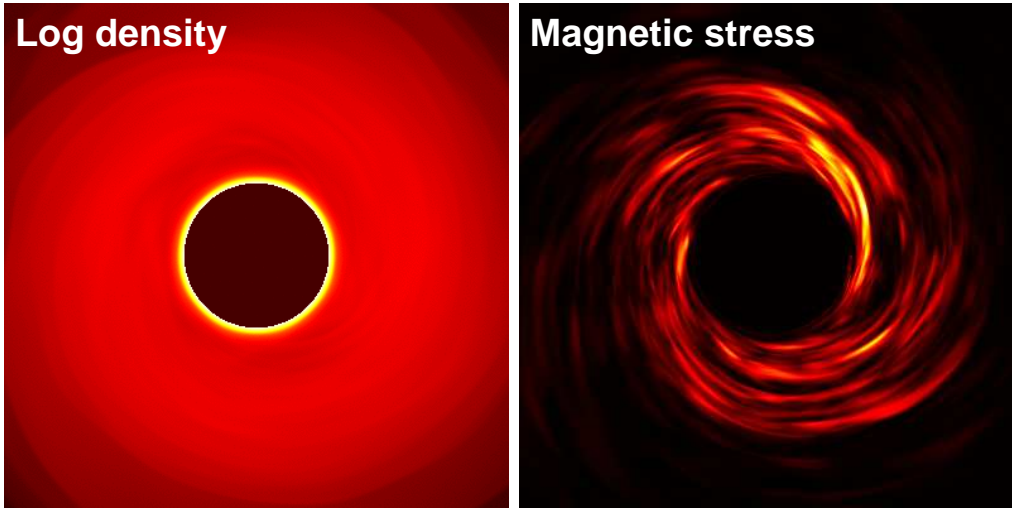


FIGURE 3. Structure in a turbulent disk accretion flow, from a global cylindrical MHD simulation (after Armitage 1998). The flow is here visualized using density (left hand panel) and the vertically averaged Maxwell stress $\langle B_r B_\phi \rangle$ (right hand panel) as tracers. In magnetically active disks, transient density and magnetic field fluctuations are present across a wide range of spatial scales.

the presence of damping, to drive a random walk in the semi-major axis of low mass planets.

To go beyond such generalities, and in particular to estimate the crossover mass between stochastic and Type I migration, we need to specify the source of turbulence in the protoplanetary disk. MHD disk turbulence (see Figure 3 for an illustration) driven by the magnetorotational instability (Balbus & Hawley 1998) provides a well-understood source of outward angular momentum transport in sufficiently well-ionized disks, and has been used as a model system for studying stochastic migration by Nelson & Papaloizou (2004) and by Laughlin, Steinacker & Adams (2004). Density fluctuations in MHD disk turbulence have a typical coherence time of the order of the orbital period, and as a consequence are able to exchange angular momentum with an embedded planet across a range of disk radii (not only at narrow resonances). The study by Nelson & Papaloizou (2004) was based on both global ideal MHD disk simulations, with an aspect ratio of $h/r = 0.07$, and local shearing box calculations. The global runs realized an effective Shakura-Sunyaev $\alpha = 7 \times 10^{-3}$, which if replicated in a real disk would be consistent with observational measures of T Tauri disk evolution (Hartmann et al. 1998). For all masses considered in the range $3 M_\oplus \leq M_p \leq 30 M_\oplus$, the *instantaneous* torque on the planet from the MHD turbulent disk exhibited large fluctuations in both magnitude and sign. Averaging over ≈ 20 orbital periods, the mean torque showed signs of converging to the Type I rate, although the rate of convergence was slow, especially for the lowest mass planets in the global runs. These properties are generally in accord with other studies of the variability of MHD disk turbulence (Hawley 2001; Winters, Balbus & Hawley 2003a). Very roughly, the Nelson & Papaloizou (2004) simulations suggest that up to $M_p \sim 10 M_\oplus$ the random walk component dominates steady Type I drift over time scales that substantially exceed the orbital period.

We caution that existing studies of the stochastic migration regime are unrealistic. Ideal MHD is not a good approximation for the protoplanetary disk at those radii where

planet formation occurs, and there may be dead zones in which MHD turbulence and angular momentum transport is highly suppressed (Gammie 1996; Sano et al. 2000; Fromang, Terquem & Balbus 2002; Salmeron & Wardle 2005). We also note that a significant random migration component, if it indeed adds to rather than supplanting steady Type I migration, does nothing (on average) to help the survival prospects of low mass planets. Nevertheless, if turbulent fluctuations (whatever their origin) do occur in the disk, the resulting random walk migration could be important for planet formation. In the terrestrial planet region, stochastic migration might deplete low mass planetary embryos that would be relatively immune to ordinary Type I migration, while simultaneously promoting radial mixing and collision of planetesimals. For giant planet formation, a significant random component to core migration would have the effect of creating large fluctuations in the planetesimal accretion rate, while also potentially acting to diffuse the planetesimal surface density. Large fluctuations in the planetesimal accretion rate favor the early onset of rapid gas accretion, and allow for the final core mass to be substantially smaller than would be expected in the case of a static core (Rice & Armitage 2003).

4. Type II migration

4.1. Conditions for the onset of Type II migration

In a viscous disk, the threshold between Type I and Type II migration can be derived by equating the time scale for Type I torques to open a gap (in the absence of viscosity) with the time scale for viscous diffusion to fill it in (Goldreich & Tremaine 1980; Papaloizou & Lin 1984). For a gap of width Δr around a planet with mass ratio $q = M_p/M_*$, orbiting at distance r_p , Type I torques can open the gap on a timescale (Takeuchi, Miyama & Lin 1996)

$$\tau_{\text{open}} \sim \frac{1}{m^2 q^2} \left(\frac{\Delta r}{r_p} \right)^2 \Omega_p^{-1}.$$

Viscous diffusion will close the gap on a time scale,

$$\tau_{\text{close}} \sim \frac{\Delta r^2}{\nu},$$

where ν , the kinematic viscosity, is usually written as $\nu = \alpha c_s^2 / \Omega_p$ (Shakura & Sunyaev 1973). Equating τ_{open} and τ_{close} , and noting that waves with $m \approx r_p/h$ dominate the Type I torque, the condition for gap opening becomes,

$$q \gtrsim \left(\frac{h}{r} \right)_p^2 \alpha^{1/2}.$$

For $h/r \simeq 0.05$ and $\alpha = 10^{-2}$, the transition (which simulations show is not very sharp) occurs at $q_{\text{crit}} \sim 2.5 \times 10^{-4}$, i.e. close to a Saturn mass for a Solar mass star. Since the most rapid Type I migration occurs when $q \approx q_{\text{crit}}$, we can also estimate a *minimum* migration time scale by combining the above expression with the time scale formula of Tanaka, Takeuchi & Ward (2002). This yields,

$$\tau_{\text{min}} \sim (2.7 + 1.1\beta)^{-1} \frac{M_*}{\Sigma r_p^2} \alpha^{-1/2} \Omega_p^{-1},$$

and is almost independent of disk properties other than the local mass.

4.2. *The rate of Type II migration*

Once a planet has become massive enough to open a gap, orbital evolution is predicted to occur on the same local time scale as the protoplanetary disk. The radial velocity of gas in the disk is,

$$v_r = -\frac{\dot{M}}{2\pi r \Sigma},$$

which for a steady disk away from the boundaries can be written as,

$$v_r = -\frac{3}{2} \frac{\nu}{r}.$$

If the planet enforces a rigid tidal barrier at the outer edge of the gap, then evolution of the disk will force the orbit to shrink at a rate $\dot{r}_p \simeq v_r$, provided that the local disk mass exceeds the planet mass, i.e. $\pi r_p^2 \Sigma \gtrsim M_p$. This implies a nominal Type II migration time scale, valid for *disk dominated migration* only,

$$\tau_0 = \frac{2}{3\alpha} \left(\frac{h}{r} \right)_p^{-2} \Omega_p^{-1}.$$

For $h/r = 0.05$ and $\alpha = 10^{-2}$, the migration time scale at 5 AU is of the order of 0.5 Myr.

In practice, the assumption that the local disk mass exceeds that of the planet often fails. For example, a $\beta = 1$ disk with a mass of $0.01 M_\odot$ within 30 AU has a surface density profile,

$$\Sigma = 470 \left(\frac{r}{1 \text{ AU}} \right)^{-1} \text{ g cm}^{-2}.$$

The condition that $\pi r_p^2 \Sigma = M_p$ gives an estimate of the radius within which disk domination ceases of,

$$r = 6 \left(\frac{M_p}{M_J} \right) \text{ AU}.$$

Interior to this radius, the planet acts as a slowly moving barrier which impedes the inflow of disk gas. Gas piles up behind the barrier – increasing the torque – but this process does not continue without limit because the interaction also deposits angular momentum into the disk, causing it to expand (Pringle 1991). The end result is to slow migration compared to the nominal rate quoted above. For a disk in which the surface density can be written as a power-law in accretion rate and radius,

$$\Sigma \propto \dot{M}^a r^b,$$

Syer & Clarke (1995) define a measure of the degree of disk dominance,

$$B \equiv \frac{4\pi r_p^2 \Sigma}{M_p}.$$

Then for $B < 1$ (the planet dominated case appropriate to small radii) the actual Type II migration rate is (Syer & Clarke 1995),

$$\tau_{II} = \tau_0 B^{-a/(1+a)}.$$

Note that with this definition of B , disk dominance extends inward a factor of a few further than would be predicted based on the simple estimate given above.

Evaluating how the surface density depends upon the accretion rate – and thereby determining the a which enters into the suppression term – requires a full model for the protoplanetary disk (not just knowledge of the slope of the steady-state surface density

profile). For the disk models of Bell et al. (1997), we find that $a \simeq 0.5$ at 1 AU for $\dot{M} \sim 10^{-8} M_{\odot} \text{yr}^{-1}$. At this radius the model with $\alpha = 10^{-2}$ has a surface density of about 200 g cm^{-2} . For a Jupiter mass planet we then have $B = 0.3$ and $\tau_{II} = 1.5\tau_0$. This is a modest suppression of the Type II rate, but the effect becomes larger at smaller radii (or for more massive planets). It slows the inward drift of massive planets, and allows a greater chance for them to be stranded at sub-AU scales as the gas disk is dissipated.

These estimates of the Type II migration velocity assume that once a gap has been opened, the planet maintains an impermeable tidal barrier to gas inflow. In fact, simulations show that planets are able to accrete gas via tidal streams that bridge the gap (Lubow, Seibert & Artymowicz 1999). The effect is particularly pronounced for planets only just massive enough to open a gap in the first place. If the accreted gas does not have the same specific angular momentum as the planet, this constitutes an additional accretion torque in addition to the resonant torque. It is likely to reduce the Type II migration rate further.

4.3. Simulations

Simulations of gap opening and Type II migration have been presented by a large number of authors, with recent examples including Bryden et al. (1999), Lubow, Seibert & Artymowicz (1999), Nelson et al. (2000), Kley, D'Angelo & Henning (2001), Papaloizou, Nelson & Masset (2001), D'Angelo, Henning & Kley (2002), D'Angelo, Kley & Henning (2003), D'Angelo, Henning & Kley (2003), Bate et al. (2003), Schäfer et al. (2004) and Lufkin et al. (2004). These authors all assumed, for simplicity, that angular momentum transport in the protoplanetary disk could be represented using a microscopic viscosity. Only a few recent simulations, by Winters, Balbus & Hawley (2003b), Nelson & Papaloizou (2004) and Papaloizou, Nelson & Snellgrove (2004), have directly simulated the interaction of a planet with a turbulent disk. For planet masses significantly above the gap opening threshold, simulations support the general scenario outlined above, while also finding:

1. **Significant mass accretion** across the gap. For planet masses close to the gap opening threshold, accretion is surprisingly efficient, with tidal streams delivering gas at a rate that is comparable to the *disk* accretion rate in the absence of a planet (Lubow, Seibert & Artymowicz 1999). It is also observed that accretion cuts off rapidly as the planet mass increases toward $10 M_J$, giving additional physical motivation to the standard dividing line between massive planets and brown dwarfs.

2. **Damping of eccentricity.** Goldreich & Tremaine (1980) noted that the evolution of the eccentricity of a planet embedded within a disk depends upon a balance between Lindblad torques, which tend to excite eccentricity, and corotation torques, which damp it. Recent analytic work (Ogilvie & Lubow 2003; Goldreich & Sari 2003; and references therein) has emphasized that if the corotation resonances are even partially saturated, the overall balance tips to eccentricity excitation. To date, however, numerical simulations (e.g. those by Papaloizou, Nelson & Masset 2001) exhibit damping for planetary mass bodies, while eccentricity growth is obtained for masses $M_p \gtrsim 20 M_J$ – in the brown dwarf regime. In a recent numerical study, however, Masset & Ogilvie (2004) present evidence that the resolution attained by Papaloizou, Nelson & Masset (2001) was probably inadequate to determine the sign of eccentricity evolution for Jupiter mass planets, which remains uncertain. Further numerical work is needed.

4.4. Comparison with statistics of extrasolar planetary systems

The estimated time scale for migration of a giant planet from 5 AU to the hot Jupiter region is of the order of a Myr. This time scale is short enough – compared to the lifetime

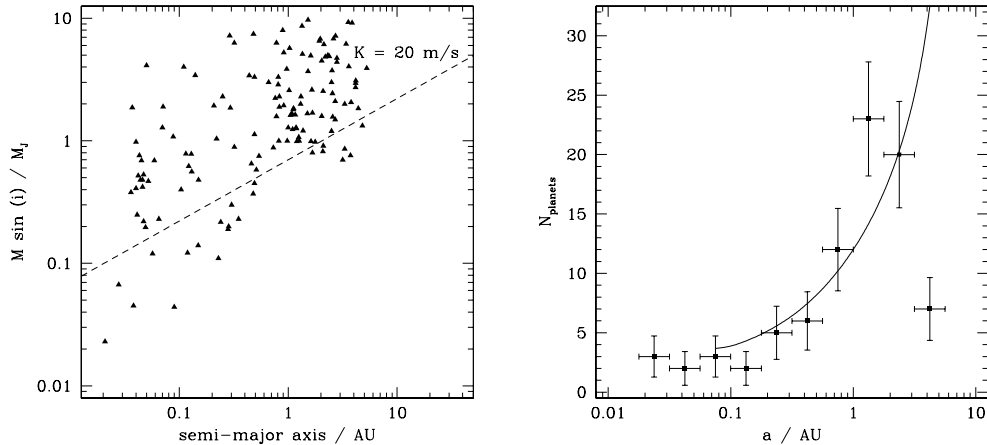


FIGURE 4. Left hand panel: the distribution of extrasolar planets discovered via radial velocity surveys in the a - $M_p \sin(i)$ plane. The dashed diagonal traces a line of equal ease of detectability – planets on circular orbits lying along lines parallel to this cause the same amplitude of stellar radial velocity variations. Right hand panel: the number of detected planets with $M_p \sin(i) > 1 M_J$ is plotted as a function of semi-major axis. The solid curve shows the predicted distribution from a pure migration model by Armitage et al. (2002). The theoretical curve is unaltered from the 2002 version except for an arbitrary normalization.

of typical protoplanetary disks – to make migration a plausible origin for hot Jupiters, while not being so short as to make large-scale migration inevitable (the latter would raise obvious concerns as to why there is no evidence for substantial migration of Jupiter itself). Having passed this initial test, it is then of interest to try and compare *quantitative* predictions of migration, most obviously the expected distributions of planets in mass and orbital radius, with observations. Models that attempt this include those by Trilling et al. (1998), Armitage et al. (2002), Trilling, Lunine & Benz (2002), Ida & Lin (2004a) and Ida & Lin (2004b). Accurate knowledge of the biases and selection function of the radial velocity surveys is essential if such exercises are to be meaningful, making analyses such as those of Cumming, Marcy & Butler (1999) and Marcy et al. (2005) extremely valuable.

Figure 4 shows how the distribution of known extrasolar planets with semi-major axis compares to the pure migration model of Armitage et al. (2002). In this model, we assumed that giant planets form and gain most of their mass at an orbital radius (specifically 5 AU) beyond where most extrasolar planets are currently being detected. Once formed, planets migrate inward via Type II migration and are either (a) swallowed by the star, or (b) left stranded at some intermediate radius by the dispersal of the protoplanetary disk. We assumed that disk dispersal happens as a consequence of photoevaporation (Johnstone, Hollenbach & Bally 1998), and that the probability of planet formation per unit time is constant over the (relatively short) window during which a massive planet can form at 5 AU and survive without being consumed by the star. Although clearly oversimplified, it is interesting that this model continues to reproduce the orbital distribution of known planets out to radii of a few AU, once selection effects have been taken into account (here, by simply ignoring low mass planets that are detectable only at small orbital radii). Moreover, it predicts that substantial *outward* migration ought to occur in disks where strong mass loss prompts an outwardly directed radial velocity in the giant planet forming region (Veras & Armitage 2004). Planets at these large radii are potentially detectable today via their effect on debris disks (Kuchner & Holman 2003).

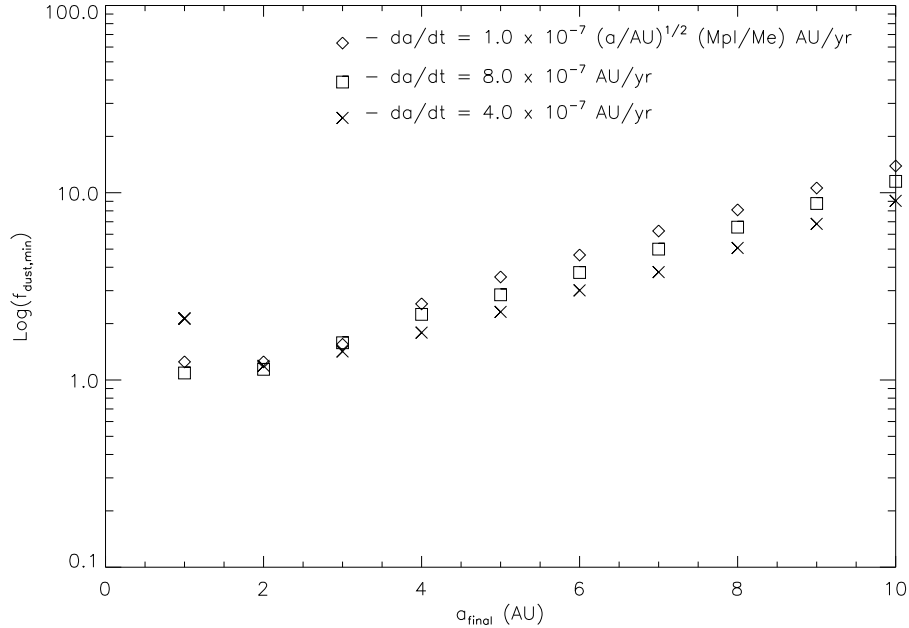


FIGURE 5. The minimum metallicity required to form a gas giant planet as a function of orbital radius, from simplified core accretion calculations by Rice & Armitage (2005). The host metallicity is expressed via the parameter f_{dust} , which is proportional to $10^{[\text{Fe}/\text{H}]}$. The models assume that dispersion in disk metallicity dominates over dispersion in either disk gas mass or disk lifetime in controlling the probability of planet formation. Type I migration of the core prior to accretion of the envelope is included, using several different prescriptions indicated by the different symbols, though none of the Type I rates are as large as the baseline rate of Tanaka, Takeuchi & Ward (2002).

Additional clues to the role of migration in forming the observed population of extra-solar planets may be possible by combining large planet samples with knowledge of the host stars' metallicity. It is now clear that the frequency of detected planets increases rapidly with host metallicity, and that the measured metallicity reflects primarily the primordial composition of the star-forming gas rather than subsequent pollution of the convective envelope (Santos, Israelian & Mayor 2004; Fischer & Valenti 2005; and references therein). The existence of such a correlation is not in itself surprising, given that the time scale for core accretion decreases quickly with increasing surface density of planetesimals. However, the sharpness of the rise in planet frequency over a fairly narrow range of stellar $[\text{Fe}/\text{H}]$ is striking, since it suggests that metallicity, rather than variations in initial gas disk mass or gas disk lifetime, may well be the single most important parameter determining the probability of giant planet formation around a particular star.

Motivated by these observations, we have investigated the use of the critical or threshold metallicity for giant planet formation as a discriminant of different planet formation models (Rice & Armitage 2005). Using simplified core accretion models similar to those of Ida & Lin (2004a), we have calculated the radial dependence of the threshold metallicity under the assumption that disks around different stars have similar initial gas masses and lifetimes (this could be replaced with the much weaker assumption that the gas mass and disk lifetime are not correlated with the metallicity). A sample result is shown in Figure 5. By definition, planets that just manage to form as the gas disk is being lost suffer little or no Type II migration, so delineating the threshold metallicity curve observationally can yield information on planet formation that is independent of

Type II migration uncertainties. We find that the most important influence on the shape of the threshold metallicity curve is probably *Type I* core migration prior to accretion of the gas envelope. When this is included we derive a monotonically rising minimum metallicity curve beyond about 2 AU. In the absence of significant core migration, the threshold metallicity is flat beyond the snow line (with a weak dependence on the surface density profile of planetesimals), and the location of the snow line may be preserved in the observed distribution of planets in the orbital radius / metallicity plane.

5. Summary

Migration via gravitational interactions between planets and the gaseous protoplanetary disk appears to be central to understanding both the formation of gas giant planets, and their early orbital evolution to yield the extrasolar planetary systems currently being observed. Although there are uncertainties in our understanding of migration, there has also been enough progress to convince us that Type I migration is probably a vital ingredient in the formation of gas giant planets via core accretion. This is trivially true using the current best estimates of the Type I migration time scale, but it would still remain important even if the rate was suppressed by as much as two orders of magnitude. On a similarly firm footing is the assertion that gas disk migration – probably in the Type II regime – is responsible for the existence of most of the hot Jupiters. Although other migration mechanisms certainly exist, it requires moderate care to *avoid* substantial orbital evolution once a planet has formed in a gas disk.

Equally interesting are the major unknowns. Does turbulence within the disk lead to random walk migration of low mass bodies, and if so, is this important for terrestrial planet formation and / or core accretion? Do corotation torques qualitatively change the behavior of migrating planets with masses just above the gap-opening threshold? Is the eccentricity of massive planets excited by the interaction with the gas disk? A positive answer to any of these questions would require substantial changes to our overall picture of planet formation. Addressing them will probably require, in part, high resolution simulations that include more of the complex physics of angular momentum transport within the protoplanetary disk.

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